Detecting bearish and bullish markets in financial time series using hierarchical hidden Markov models

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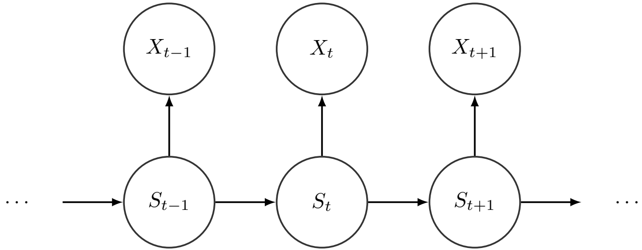
In a nutshell: Bearish and bullish markets

In a nutshell: The hidden Markov model



- $(X_t)_t$ – observed state-dependent process, $X_t \mid S_t = i \sim f_i$
- $(S_t)_t$ – state process, e.g. state space $= \{\text{bullish}, \text{bearish}, \text{correction}\}$

What we are going to talk about?

Why hierarchical hidden Markov models for financial time series data?

How to estimate such models?

How to decode the hidden states?

Model results for the DAX

Model results for the Goldman Sachs Group stock

How to perform model checking?

What's next?

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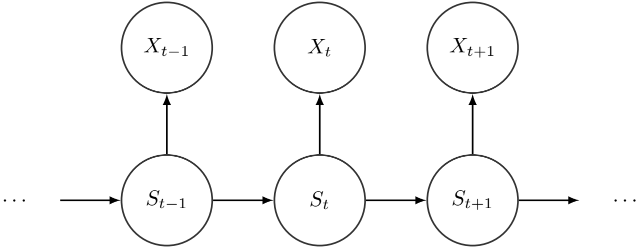
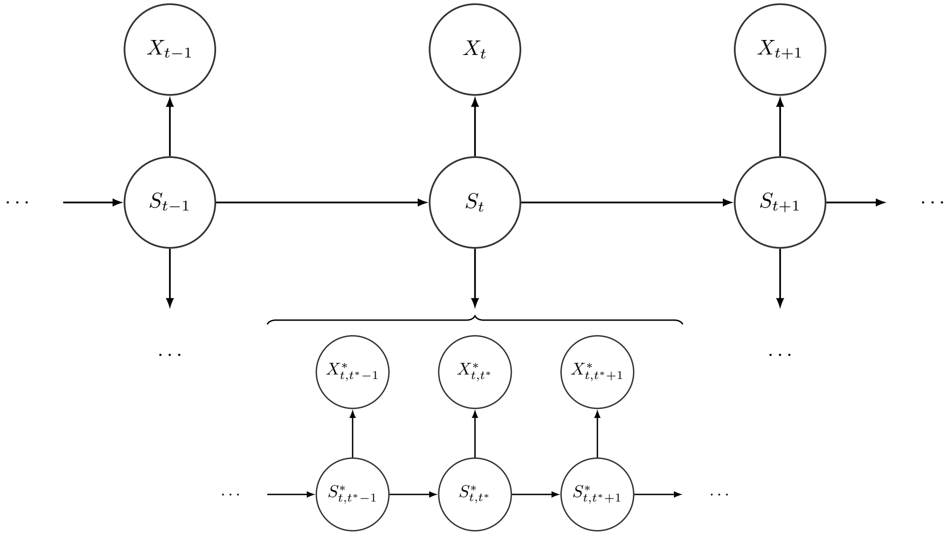


- successful results, leading to trading strategies that outperform e.g. buy-and-hold
- model does not capture short and long term trends jointly


X_{t-1}
X_t
X_{t+1}
$\cdots$
S_{t-1}
S_t
S_{t+1}
$\cdots$
$\cdots$
$\cdots$
X^*_{t,t^*-1}
X^*_{t,t^*}
X^*_{t,t^*+1}
$\cdots$
S^*_{t,t^*-1}
S^*_{t,t^*}
S^*_{t,t^*+1}
$\cdots$

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Likelihood of the i-th fine-scale hidden Markov model with the model parameters $\theta^{*(i)} = (\delta^{*(i)}, \Gamma^{*(i)}, (f^{*(i,k)})_k)$ on the t-th chunk $(X_{t,t^*})_{t^*}$ of fine-scale observations

$$\mathcal{L}^{HMM}(\theta^{*(i)} \mid (X^*_{t,t^*})_{t^*}) = \sum_{S^*_{t,1},\ldots,S^*_{t,T^*}=1}^{N^*} \left(\prod_{t^*=1}^{T^*} f^{*(i,S^*_{t,t^*})}(X^*_{t,t^*}) \right) \left(\delta^{*(i)}_{S^*_{t,1}} \prod_{t^*=2}^{T^*} \gamma^{*(i)}_{S^*_{t,t^*-1} S^*_{t,t^*}} \right)$$

Complexity: exponential

Maximum likelihood estimation

fine-scale forward probabilities

$$\alpha_{k,t^*}^{*(i)} = f^{*(i)}(X_{t,1}^*, \ldots, X_{t,t^*}^*, S_{t,t^*}^* = k),$$

$$\mathcal{L}^{HMM}(\theta^{*(i)} \mid (X_{t,t^*}^*)_{t^*}) = \sum_{k=1}^{N^*} \alpha_{k,T^*}^{*(i)}.$$

$$\begin{aligned}
\alpha_{k,1}^{*(i)} &= \delta_k^{*(i)} f^{*(i,k)}(X_{t,1}^*), \\
\alpha_{k,t^*}^{*(i)} &= f^{*(i,k)}(X_{t,t^*}^*) \sum_{j=1}^{N^*} \gamma_{jk}^{*(i)} \alpha_{j,t^*-1}^{*(i)}, \;\; t^* = 2, \ldots, T^*.
\end{aligned}$$

Complexity: linear

Problems in the numerical maximization

- parameter contraints $\rightarrow$ bijective transformation
- numerical underflow $\rightarrow$ logarithm
- local maxima $\rightarrow$ repeated numerical search

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Viterbi algorithm

We are interested in

$$\arg\max_{S_1,\ldots,S_T} f(S_1,\ldots,S_T \mid X_1,\ldots,X_T) = \arg\max_{S_1,\ldots,S_T} f(S_1,\ldots,S_T,X_1,\ldots,X_T),$$

which we derive from

$$\xi_{i,t} = \max_{S_1,\ldots,S_{t-1}} f(S_1,\ldots,S_{t-1},S_t = i, X_1,\ldots,X_t),$$

which can in turn be calculated recursively via

$$\xi_{i,1} = \delta_i f^{(i)}(X_1),$$
$$\xi_{i,t} = \max_j \left(\xi_{j,t-1}\gamma_{ji}\right) f^{(i)}(X_t).$$

Then

$$\hat{S}_T = \arg\max_i \xi_{i,T},$$
$$\hat{S}_t = \arg\max_i \xi_{i,t}\gamma_{i\hat{S}_{t+1}}.$$

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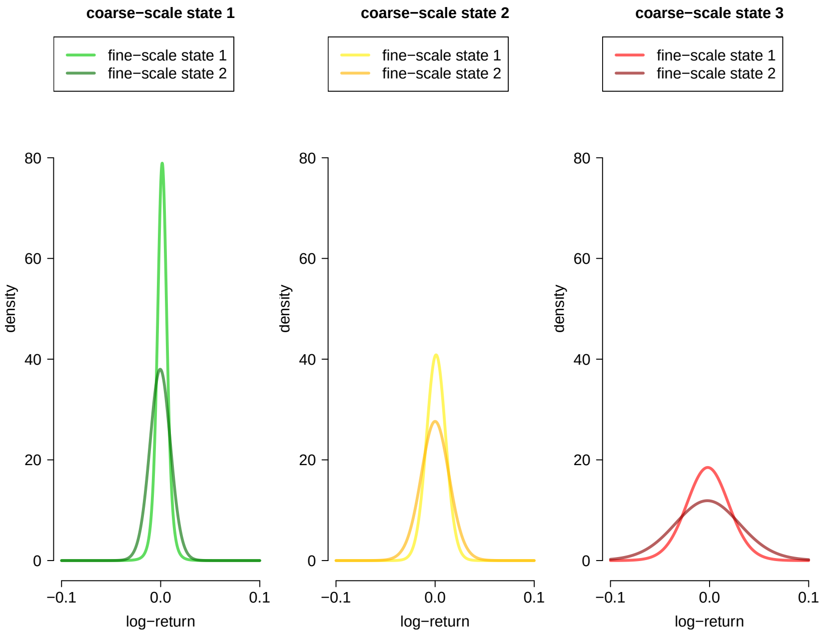
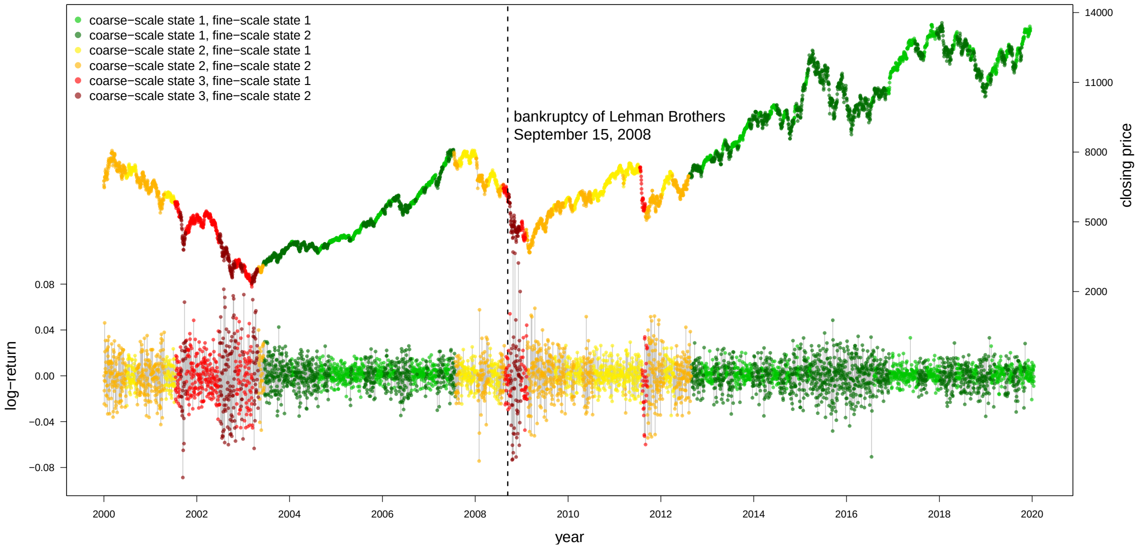
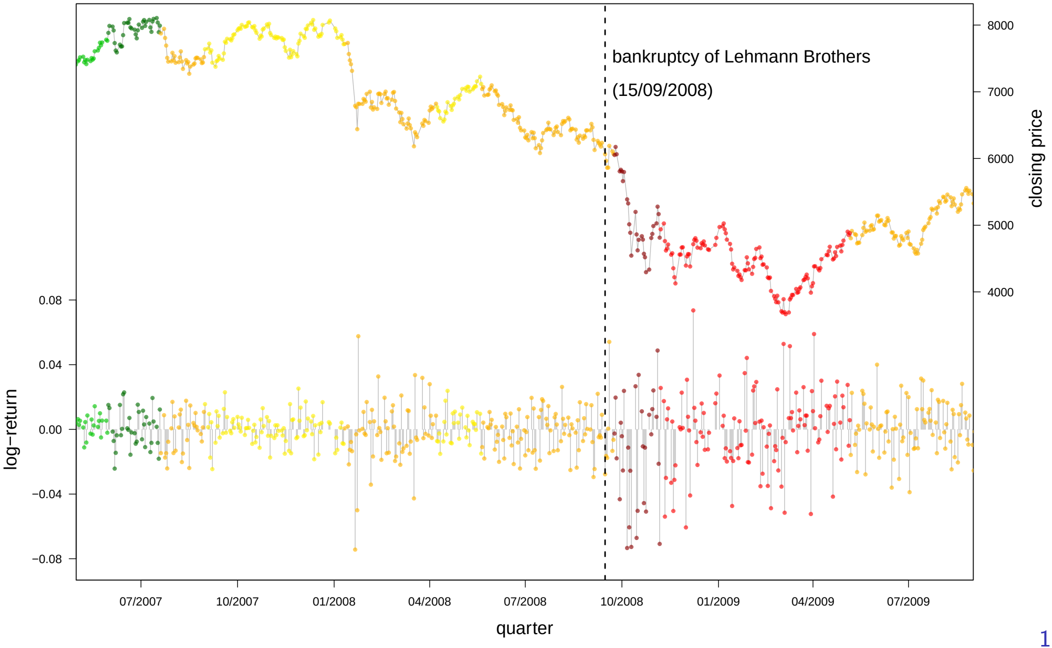


coarse−scale state 1
fine−scale state 1
fine−scale state 2
coarse−scale state 2
fine−scale state 1
fine−scale state 2
coarse−scale state 3
fine−scale state 1
fine−scale state 2
density
0
20
40
60
80
−0.1
0.0
0.1
log−return


coarse−scale state 1, fine−scale state 1
coarse−scale state 1, fine−scale state 2
coarse−scale state 2, fine−scale state 1
coarse−scale state 2, fine−scale state 2
coarse−scale state 3, fine−scale state 1
coarse−scale state 3, fine−scale state 2
bankruptcy of Lehman Brothers
September 15, 2008
14000
11000
8000
5000
2000
closing price
0.08
0.04
0.00
−0.04
−0.08
log−return
2000
2002
2004
2006
2008
2010
2012
2014
2016
2018
2020
year



bankruptcy of Lehmann Brothers
(15/09/2008)
8000
7000
6000
5000
4000
closing price
0.08
0.04
0.00
−0.04
−0.08
log−return
07/2007
10/2007
01/2008
04/2008
07/2008
10/2008
01/2009
04/2009
07/2009
quarter

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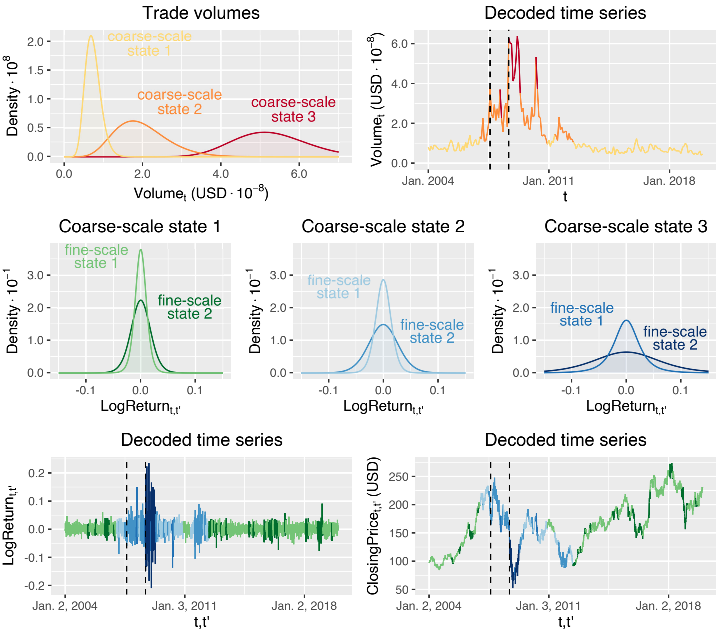
Trade volumes
coarse-scale state 1
coarse-scale state 2
coarse-scale state 3
Density · 10^8
Volume$_t$ (USD · 10^{-8})
Decoded time series
Volume$_t$ (USD · 10^{-8})
Jan. 2004
Jan. 2011
Jan. 2018
t
Coarse-scale state 1
fine-scale state 1
fine-scale state 2
Density · 10^{-1}
LogReturn$_{t,t'}$
Coarse-scale state 2
fine-scale state 1
fine-scale state 2
Density · 10^{-1}
LogReturn$_{t,t'}$
Coarse-scale state 3
fine-scale state 1
fine-scale state 2
Density · 10^{-1}
LogReturn$_{t,t'}$
Decoded time series
LogReturn$_{t,t'}$
Jan. 2, 2004
Jan. 3, 2011
Jan. 2, 2018
t,t'
Decoded time series
ClosingPrice$_{t,t'}$ (USD)
Jan. 2, 2004
Jan. 3, 2011
Jan. 2, 2018
t,t'

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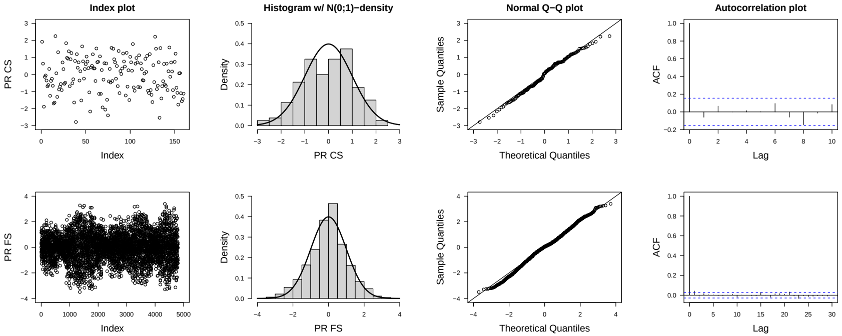
Pseudo Residuals

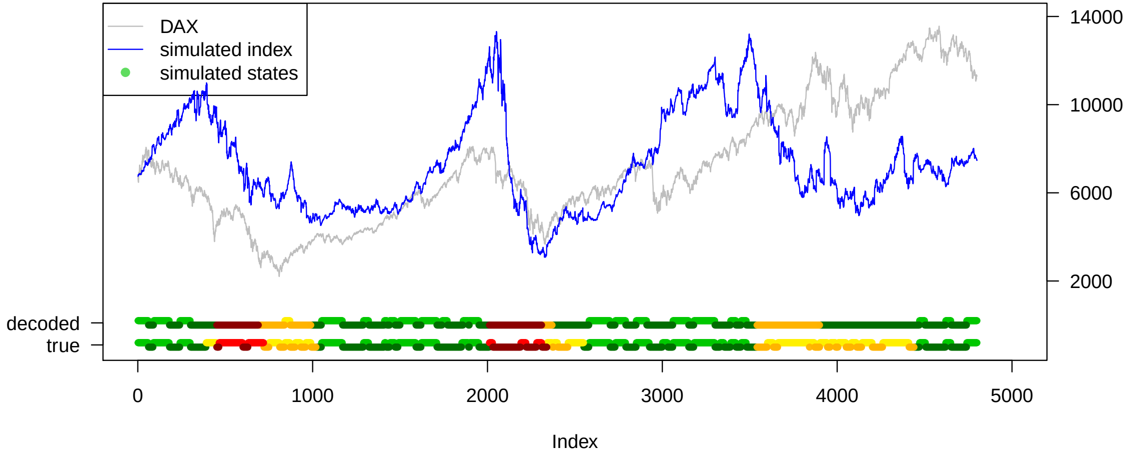
$$X_t \text{ has (invertible) distribution function } F_{X_t} \Rightarrow Z_t = \Phi^{-1}(F_{X_t}(X_t)) \sim \mathcal{N}(0,1)$$

Check:

- $Z_t \overset{a}{\sim} \mathcal{N}(0,1)$?
- $\mathrm{Cov}(Z_t, Z_{t+h}) \approx 0$?



Bootstrapping



André Kostolany (1906-1999)

«Think long-term. Prices rise or fall over months and years. There is no need to let yourself be driven crazy by short-term fluctuations.»

Thanks for your attention!